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LETTER TO THE EDITOR

On the linearity of the parametrised post-Newtonian metric

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Abstract. The coordinate freedom at the post-Newtonian level is used to obtain a parametrised post-Newtonian metric from which the dominating nonlinear term is absent for all gravitational theories within this framework. This absence makes explicit the fact that gravitational nonlinearities are not observable in the post-Newtonian approximation.

Nonlinearities in a metric may reflect real physical nonlinearities in the gravitational field equations. They may also, however, be the result of the coordinate system chosen, as is stressed by Deser and Laurent (1973). The Schwarzschild solution in general relativity provides an example of this. In Eddington's form (Eddington 1924) the metric is fully linear in $1/r$, while in Schwarzschild coordinates it is linear only in the post-Newtonian approximation. In isotropic coordinates, which are used by Will, Nordvedt and others in the parametrised post-Newtonian (PPN) formalism (see e.g. Will and Nordvedt 1972, Will 1974), the Schwarzschild solution is already nonlinear in this approximation.

It was shown by Deser and Laurent (1973) that a general stationary spherically symmetric gravitational field can be described by a linear metric at the post-Newtonian level. Such a metric has the advantage of not obscuring the origin of nonlinearities, but of making explicit the fact that gravitational nonlinearities are not measurable in the post-Newtonian approximation. In this letter we generalise this result to a general gravitational field. It is found that the dominating nonlinear term (the one in g_{00} which is proportional to the square of the Newtonian potential) can be gauged to zero by using the coordinate freedom at the post-Newtonian level. The parameters used in this linear PPN metric are the same as in the conventional metric. This makes it possible to use all the results from the old formalism (such as parameter values for a specific theory or observational limits on the parameters).

The conventional PPN metric in isotropic coordinates is (Will 1974)

$$\begin{aligned} g_{00} &= 1-2U+2\beta U^2-(2\gamma+2+\alpha_3+\zeta_1)\Phi_1+\zeta_1\mathscr{A} \\ &\quad -2[(3\gamma-2\beta+1+\zeta_2)\Phi_2+(1+\zeta_3)\Phi_3+3(\gamma+\zeta_4)\Phi_4] \\ &\quad +(\alpha_1-\alpha_2-\alpha_3)w^2U+\alpha_2 w^\alpha w^\beta U_{\alpha\beta}-(2\alpha_3-\alpha_1)w^\alpha V_\alpha+O_6 \\ g_{0\alpha} &= \tfrac{1}{2}(4\gamma+3+\alpha_1-\alpha_2+\zeta_1)V_\alpha+\tfrac{1}{2}(1+\alpha_2-\zeta_1)W_\alpha+\tfrac{1}{2}(\alpha_1-2\alpha_2)w^\alpha U+\alpha_2 w^\beta U_{\alpha\beta}+O_5 \\ g_{\alpha\beta} &= -(1+2\gamma U)\delta_{\alpha\beta}+O_4. \qquad (1) \end{aligned}$$

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The coordinate transformation

$$y^\alpha = x^\alpha - \varepsilon\chi_{,\alpha} \qquad y^0 = x^0 \tag{2}$$

where

$$\chi = -\int \rho'|\boldsymbol{x} - \boldsymbol{x}'|\,\mathrm{d}\boldsymbol{x}'$$

gives

$$\begin{aligned} g'_{00}(y^k) &= g_{00}(x^k) + O_6 \\ g'_{0\alpha}(y^k) &= g_{0\alpha}(x^k) - \varepsilon\chi_{,\alpha 0} + O_5 \\ g'_{\alpha\beta}(y^k) &= g_{\alpha\beta}(x^k) - 2\varepsilon\chi_{,\alpha\beta} + O_4. \end{aligned} \tag{3}$$

Replacing x^k by y^k in (3) gives negligible errors everywhere, except in the Newtonian part of g_{00}. There one finds (Karlhede 1978)

$$U(\boldsymbol{x}) = U(\boldsymbol{y}) + \varepsilon U^2 - \varepsilon\Phi_2 + \varepsilon\Phi_w \tag{4}$$

where Φ_w is the term introduced by Will in connection with Whitehead's theory. The PPN metric then becomes

$$\begin{aligned} g'_{00} = 1 &- 2U + 2(\beta - \varepsilon)U^2 - (2\gamma + 2 + \alpha_3 + \zeta_1)\Phi_1 + \zeta_1\mathscr{A} \\ &- 2[(3\gamma - 2\beta + 1 + \zeta_2 - \varepsilon)\Phi_2 + (1 + \zeta_3)\Phi_3 + 3(\gamma + \zeta_4)\Phi_4] \\ &- 2\varepsilon\Phi_w + (\alpha_1 - \alpha_2 - \alpha_3)w^2 U + \alpha_2 w^\alpha w^\beta U_{\alpha\beta} - (2\alpha_3 - \alpha_1)w^\alpha V_\alpha + O_6 \\ g'_{0\alpha} = \tfrac{1}{2}&(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\varepsilon)V_\alpha + \tfrac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\varepsilon)W_\alpha \\ &+ \tfrac{1}{2}(\alpha_1 - 2\alpha_2)w^\alpha U + \alpha_2 w^\beta U_{\alpha\beta} + O_5 \\ g'_{\alpha\beta} = -&\delta_{\alpha\beta} - 2(\gamma - \varepsilon)U\delta_{\alpha\beta} - 2\varepsilon U_{\alpha\beta} + O_4. \end{aligned} \tag{5}$$

The term Φ_w should be introduced from the beginning among the possible terms in g_{00}. That it fulfils the requirements for this is perhaps most easily seen from the relation (Karlhede 1978)

$$2\Phi_2 + 2\Phi_w = -\frac{1}{\pi}\int \frac{(U'U'_{\alpha\beta})_{,\alpha\beta}}{|\boldsymbol{x} - \boldsymbol{x}'|}\,\mathrm{d}\boldsymbol{x}'. \tag{6}$$

Choosing $\varepsilon = \beta$ makes U^2, the dominating nonlinear term, disappear. There are of course other terms which are nonlinear in the source distribution, such as $\Phi_2 = \int (\rho' U'/|\boldsymbol{x} - \boldsymbol{x}'|)\,\mathrm{d}\boldsymbol{x}'$. The leading parts of these, however, are proportional to $1/r$. For general relativity $\varepsilon = \beta$ gives Schwarzschild coordinates, since $\gamma = \beta = 1$ in this case.

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References

Deser S and Laurent B 1973 *Astron. Astrophys.* **25** 327

Eddington A 1924 *Nature* **113** 192

Karlhede A 1978 *University of Stockholm, Institute of Physics Report* USIP 78–13

Will C M 1974 in *Experimental Gravitation: Proc. Course 56 of the Int. School of Physics 'Enrico Fermi'* ed B Bertotti (New York: Academic) pp 1–110

Will C M and Nordvedt K Jr 1972 *Astrophys. J.* **177** 757